SYLLABUS FOR SUBSIDIARY MATHEMATICS (SMT)

PAPER-I : Theory (100 marks)

BLOCK-1

Group-A : Classical Algebra

UNIT 1 : Complex Numbers **Marks : 25**

De Moiver's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a^z for $a \neq 0$. Inverse circular and Hyperbolic functions.

UNIT 2 : Polynomials

Fundamental Theorem of Classical Algebra (Statement and application only). Polynomials with real coefficients. The nth degree polynomial equation has exactly n roots. Nature of roots of an equation (Surd or Complex roots occur in pairs). Statement of Descarte's Rule of signs and its applications.

(i) For any two real numbers a and b, if the polynomial $f(a)$. $f(b) < 0$ ($f(a)$. $f(b) > 0$) the equation $f(x) = 0$ has an odd (even) number of real roots between a and b.

(ii) Statements of Rolle's Theorem and its direct applications.

Relation between roots and coefficients. Symmetric functions of roots, Transformations of a polynomial equation. Cardan's method of solution of a cubic equation.

UNIT 3 : Matrices and Determinants

Matrices of Real Numbers— Equality of matrices. Addition of matrices. Multiplication of a matrix by a scalar. Multiplication of matrices and its Associative property. Transpose of a matrix — its properties.

Determinants up to the third order. Properties, Cofactor and Minor. Product of two determinants. Adjoint, Symmetric and Skew-symmetric determinants. Solutions of linear equations with not more than three variables by Cramer's Rule.

UNIT 4 : Elementary operations and System of Linear Equations

Inverse of a non-singular square matrix. Symmetric and Skew - symmetric matrices. Scalar matrix. Orthogonal matrix. Elementary operations on matrices.

Rank of a matrix, Determination of rank. Consistency and solution of a system of linear of equations with not more than 3 variables by matrix method.

Group-B : Modern Algebra

UNIT 5 : Set, Relation and Mapping **Marks : 25**

UNIT 6 : Group

Binary relations, Groupiod, Semigroup, Monogroup, Definition and examples of group from various branches (examples from number system, roots of unity, 2×2 real matrices, non-singular real matrices of a fixed order). Elementary properties using the definition of group. Definition and examples of sub groups, cyclic groups. Permutation - even and odd permutation, group of permutation.

UNIT 7 : Ring-Field

Definition and examples of ring, sub-ring. Integral Domain. Divisior of zero. Field, every field is an integral domain, Sub-field of a field.

UNIT 8 : Eigen value and Eigen vectors

Characteristic equation of a square matrix of order not more than three, determination of eigen values and eigen vectors. Cayley-Hamilton theorem statement and illustration

BLOCK-2

Group-A : Analytical Geometry of two and three dimensions

Two Dimensions : **Marks : 35**

UNIT 1 : Transformations of Rectangular axes, Translation, Rotation and their combinations. Invariants.

UNIT 2 : Condition that the general equation of 2nd degree in x and y may represent two straight lines. Point of intersection of two intersecting straight lines. Angle between two lines given by $ax^2 + 2hxy + by^2 = 0$. Equation of bisectors of two intersecting straight lines. Equation of two lines joining the origin to the points in which a line meets a conic.

General equation of second degree in x and y : Reduction to canonical forms. Classification of conics.

UNIT 3 : Poles and Polars

Equations of pair of tangents from an external point, chord of contact, poles and polars in case of General conic: Particular cases for Parabola, Ellipse, Circle, Hyperbola.

UNIT 4 : Polar Equations

Polar equation of straight lines and circles. Polar equation of a conic referred

to a focus as pole. Equation of chord joining two points. Equations of tangent and normal.

Three Dimensions :

UNIT 5 : *Rectangular Cartesian coordinates:* Distance between two points. Division of a line segment in a given ratio. Direction cosines and direction ratios of a straight line. Projection of a line segment on another line. Angle between two straight lines.

UNIT 6 : *Plane and Straight Lines :* Equation of a Plane, General form. Intercept and Normal form. Angle between two planes. Distance of a point from a plane. Bisectors of angles between two intersecting planes.

UNIT 7 : Equations of Straight line, General and symmetric form. Distance of a point from a line. Coplanarity of two straight lines. Shortest distance between two skew-lines.

UNIT 8 : **Sphere and Cone :** Sphere and its tangent plane, Right circular cone.

Group-B : Vector Algebra

UNIT 9 : **Vector** Marks : 15

Collinear and coplanar vectors.

UNIT 10 : Multipication of vectors and Geometrical Application

Scalar and vector product of two vectors. Scalar triple product of three vectors and its geometrical interpretation, simple application to geometry. Vector equations of straight lines and planes.

PAPER-II : Theory (100 marks)

BLOCK-1

Differential Calculus

Marks : 50

UNIT 1 : Real Number

Rational numbers. Geometrical representations. Irrational numbers. Real numbers represented as a point on line — Linear Continuum. Acquaintance with basic properties of real number (No deduction or proof is included).

UNIT 2 : Sequence

Definition of Sequence, bounded sequence and monotone sequence. Limit of a sequence. Statements of limit theorems. Concept of convergence and divergence of monotonic sequences— applications of the theorems, in particular definition of *e*. Statement of Cauchy's general principle of convergence and its application.

UNIT 3 : Infinite series

Infinite series of constant terms — Convergence and Divergence (definitions). Cauchy's principle as applied to infinite series (application only). Series of positive terms-Statements of Comparison test, D. Alembert's Ratio test, Cauchy's *n*th root test and Raabe's test—Applications. Alternating series — Statement of Leibnitz test and its applications.

UNIT 4 : Functions, Limit, Continuity

Real-valued functions : defined on an interval, Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval.

Acquiantance (no proof) of limits with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotonic function and its continuity.

UNIT 5 : Derivative including successive derivetive

Derivative — its geometrical and physical interpretation. Sign of derivative — Monotonic increasing and decreasing functions. Relation between continuity and derivability. Differential — application in finding approximation.

Successive derivative-— Leibnitz's Theorem and its application.

UNIT 6 : Rolle's theorem, Mean value theorems and L' Hospital's Rule

Statement of Rolle's Theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy. Indeterminate Forms : L Hospital's Rule : Statement and problems only;

UNIT 7 : Taylor's theorem, Maxima and Minima

Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders. Taylor's and Maclaurin's Infinite series for functions like e^x, $\sin x$, $\cos x$, $(1+x)^n$, $\log(1+x)$ [with restrictions wherever necessary] Application of the principle of Maxima and Minima for a function of single variable in geometrical, physical and other problems.

UNIT 8 : Differential calculus of multi variable function

Functions of two and three variables— Their geometrical representations. Limit and Continuity (definitions only) for funcitions of two variables. Partial derivatives Chain rule. Exact differentials (emphasis on solving problems only). Functions of two variables — Successive partial derivatives: Statement of Schwarz's Theorem on commutative property of mixed derivatives. Euler's Theorem on homogeneous function of two and three variables. Maxima and minima of functions of not more than three variables. Lagrange's Method of undetermined multiplier. Implicit function of function of two variables (existence assumed) and derivative.

UNIT 9 : Tangents of a curve and Rectilinear asymptotes

Tangents and Normals of a curve, Pedal equation and Pedal of a curve. Rectilinear Asymptotes in Cartesian coordinate system.

UNIT 10 : Curvature of and Envelops

Curvature of plane curves. Envelope of family of straight lines and of curves. Definitions and examples'of singular points (viz. Node, Cusp, Isolated point).

BLOCK-2

Group-A : Integral Calculas

30 Marks

UNIT 1 : Integration

Integration of the form :

$\int \frac{dx}{a+b\cos x}, \int \frac{l\sin x+m\cos x}{q\sin x+p\cos x}dx$ and Integration of Rational functions.

UNIT 2 : Evaluation of definite Integrals

UNIT 3 : Integration as the limit of a sum (with equally spaced as well as unequal intervals).

UNIT 4 : Reduction formulae $\int \sin^n x dx \cos^n x dx, \int \frac{\sin^m x}{\cos^n x} dx, \int \tan^n x dx$ associated problems (where m and n are non-negative integers).

UNIT 5 : Definition of Improper Integrals : Statements of (i) μ-test, (ii) Comparison test (Limit form excluded) — Simple problems only. Use of Beta and Gamma functions (convergence and important relations being assumed).

UNIT 6 : Double integral.

UNIT 7 : Applications : Rectification, Quadrature, Volume and Surface areas of solids formed by revolution of plane curve and areas — Problems only.

Group-B : Ordinary Differential Equations

20 Marks

UNIT 8 : First order Equations

Order, degree and solution of an ordinary differential equation (ODE) in presence of arbitrary constants. Formation of ODE. Solution of general First order equations by,

(i) Seperation of Variables

(ii) Homogeneous equations and equations reducible to homogeneous forms.

(iii) Exact equations and those reducible to Exact equation.

(iv) Euler's and Bernoulli's equations (Linear).

(v) Clariaut's Equations : General and Singular solutions.

UNIT 9 : Second order linear equations :

Second order linear differential equations with constant coefficients. Euler's Homogeneous equations.

UNIT 10 : Geometric Application

Simple applications : Orthogonal Trajectories.

PAPER-III : Theory (100 marks)

BLOCK-1

Group-A : Numerical Analysis

20 Marks

UNIT 1 : Approximation of Numbers and Error Analysis : Approximate numbers, Significant digits, Rounding off numbers. Error—Absolute, Relative and Percentage.

UNIT 2 : Operators : Definitions and properties of the operators Δ, ∇ and E ; relations between those operators

UNIT 3 : Interpolation : Interpolation with Equispaced arguments – Difference Tables, – Deduction of Newton's Forward Interpolation Formula, Remainder term (expression only). Newton's Backward Interpolation Formula (statement only) with remainder term.

Interpolation with Unequally spaced arguments – Lagrange's Interpolation Formula (statement only). Numerical problems on Interpolation with both equispaced and unequally spaced arguments.

UNIT 4 : Numerical Integration : Trapezoidal and Simpson's 1/3rd formula (statement and examples). Problems on Numerical Integration.

UNIT 5 : Solution of Numerical Equation : To find a real root of an algebric or transcendental equation. Location of root (Tabular method), Bisection method, Newton-Raphson method with geometrical significance, Numerical problems.

(Note : Emphasis should be given on problems)

Group-B : Analytical Dynamics

30 Marks

UNIT 6 : Impulse and impulsive forces.

UNIT 7 : Motion in a straight line under variable forces, damped forces and damped forced vibration, motion under inverse square law. Velocity and accelerations of a particle in cartesian and polar co-ordinates. Tangential and normal accelerations, circular motion.

UNIT 8 : Motion in a plane : equations of motion in cartesian and polar coordinates, central orbits, escape velocity.

BLOCK-2

Linear Programming and Game Theory

50 Marks

UNIT 1 : Definition of L.P.P. Formation of L. P. P. from daily life involving inequations. Graphical solution of L. P. P.

UNIT 2 : Basic solutions and Basic feasible solutions (BFS),
Matrix formulation of L. P. P., Degenerate and Non-degenerate B.F.S., Hyperplane, Convex set, Cone, Extreme points, Convex hull and convex polyhedron, Supporting and Separating hyperplane.
The collection of all feasible solutions of an L. P. P. constitutes a convex set; The extreme points of the convex set of feasible solutions correspond to its B.F.S. and conversely. The objective function has its optimal value at an extreme point of the convex polyhedron generated by the set of feasible solutions, (the convex polyhedron may also be unbounded). In the absence of degeneracy, if the L. P. P. admits of an optimal solution, then at least one, B. F. S. must be optimal. Reductions of a F. S. to a B. F. S.

UNIT 3 : Slack and surplus variables. Standard form of L. P. P., Theory of simplex method. Feasibility and optimality conditions.

UNIT 4 : The algorithm. Two phase method. Degeneracy in L.P.P. and its resolution.

UNIT 5 : Duality Theory. The dual of the dual is the primal. Relation between the objective values of dual and primal problems. Relation between their optimal values. Complementary slackness. Duality and simplex method and their application.

UNIT 6 : Transportation and Assignment problems. Mathematical justification for optimatility criterion. Hungarina method. Travelling Salesman problem.

UNIT 7 : Concept of Game. Rectangular games. Pure strategy and Mixed strategy. Saddle point tand its existence. Optimal strategy and value of the game, Necessary and sufficient condition for a given strategy to be optimal. Concept of Dominance, Fundamental Theorem of Rectangular games, Algebric method, Graphical method and Dominance method of solving Rectangular games. Inter-relation between the theory of Games and L.P.P.