

NSOU NEP-MATHEMATICS (NMT) SYLLABUS

Academic Level	Semester	Major (60/80)	Minor (24/32)	MDC (9)	SEC (9)	Field work/Project/Internship(4/12)
	I	5CC-MT-01 Algebra-I	NEC-MT-01 - Basic Algebra	NMD-MT-01 Statistical Techniques	NSE-MT-01 Analytical Geometry	
	II	5CC-MT-02 Real Analysis-I	NEC-MT-02 Fundamentals of Analytical Geometry		NSE-MT-02 Linear Programming	
Level- 5	Exit 1	UG Certified Programme				
	III	6CC-MT-03- Numerical Analysis	NEC-MT-3- Introductory Calculus		NSE-MT-03- Computer Programming	
		6CC-MT-04- Calculus				
	IV	6CC-MT-05- Differential Equation	NEC-MT-04 Basic Linear Programming			
		6CC-MT-06- Riemann Integration and Series of Functions				

		6CC-MT-07- Group Theory				
Level- 6	Exit 2	UG Diploma Programme				
	V	7CC-MT-08 Multivariate Calculus and PDE-I				
		7CC-MT-09 Ring Theory and Linear Algebra				
		7CC-MT-09 Mechanics				
		7CC-MT-10 Probability and Statistics				
	VI	7CC-MT-11 Complex analysis & Metric space				
		7CC-MT-12 - Real analysis-II				
		7CC-MT-13 Dynamical Systems				
		7CC-MT-15 - General Topology				
Level -7	Exit 3	UG Degree programme				
	VII	8CC-MT-16 Algebra-II			Research Methodology(4)	Seminar/Presentation/ Internship/Community engagement /Value Added Activity (4)

		8CC-MT-17- PDE-II			
		8CC-MT-18 Functional Analysis			
		8CC-MT-19: Integral Eq. & Integral Transform			
	V III	8CC-MT-20 -Geometry of Manifolds			
		8CC-MT-21 Principle of Mechanics			
		8CC-MT-22 Graph Theory			Project/Dissertation (12)
		8CC-MT-23 Adv. ODE & Special Functions			
Level -8	Exit 4	UG Degree honours/Research Programme			

Table-1: Format applicable for UG Programmes in Mathematics

Detailed Syllabus

Major Courses

5CC-MT-01: Algebra-I

Polar representation of complex numbers, n^{th} roots of unity, De Moivre's theorem for rational indices and its applications. Exponential, logarithmic, trigonometric and hyperbolic functions of complex number.

General properties of equations, Descartes's rule of signs, relation between roots and coefficients, transformation of equations, cubic and biquadratic equations.

The inequality involving $AM > GM > HM$, Cauchy-Schwartz inequality.

Inverse of a matrix, Rank of a matrix,

Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation $Ax=b$, solution sets of linear systems, applications of linear systems, linear independence. characteristic equation of a matrix, Eigen value of a matrix, Eigen vectors, Cayley-Hamilton theorem.

Equivalence relations, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

5CC-MT-02: Real Analysis-I

Algebraic and Order Properties of R , Neighborhood of a point in R , Idea of countable sets, uncountable sets and uncountability of R . Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Supremum and Infimum, The Completeness Property of R , The Archimedean Property, Density of Rational (and Irrational) numbers in R , Intervals. Limit points of a set, isolated points, Illustrations of Bolzano-Weierstrass theorem for sets.

Limits of functions, sequential criterion for limits, divergence criteria. Limit theorems, one sided limits. Infinite limits and limits at infinity. Continuous functions. Algebra of continuous functions. Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem.

Sequences, Bounded sequence, Convergent sequence, Limit of a sequence. Limit Theorems, Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano Weierstrass Theorem for Sequences. Cauchy sequence, Cauchy's Convergence Criterion. sequential criterion for continuity and discontinuity

Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's n^{th} root test, Integral test, Alternating series, Leibniz test, Absolute and Conditional convergence, Power series, radius of convergence,

6CC-MT-03: Numerical Analysis

(Use of Scientific Calculator is allowed.)

Algorithms, Convergence, Errors: Relative, Absolute, Round off, Truncation.

Transcendental and Polynomial equations: Bisection method, Secant method. Regular-falsi method, Newton-Raphson method, Rate of convergence of these methods.

System of linear algebraic equations: Gaussian elimination and Gauss-Jordan methods. Gauss-Jacobi, Gauss-Siedel and SOR iterative methods and their convergence analysis.

Interpolation: Lagrange's and Newton's methods (forward difference, backward difference and central difference interpolation), error bounds, finite difference operators.

Numerical differentiation.

Numerical Integration: Newton cotes formula, trapezoidal rule, Simpson's rule, Weddle's rule.

Computer Language: Concept of programming languages, Machine language, Assembly language, High-level language, Interpreter, Compiler, Source and Object programs.

Number Systems: Binary, decimal, octal and hexadecimal number systems and their conversions.

6CC-MT-04: Calculus

Differentiability of a function at a point and in an interval, algebra of differentiable functions. Relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorem, intermediate value property of derivatives, Darboux's theorem. Applications of mean value theorem to inequalities and approximation of polynomials, Taylor's theorem to inequalities.

Cauchy's mean value theorem. Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder, application of Taylor's theorem to convex functions, relative extrema. Taylor's series and Maclaurin's series expansions of exponential and trigonometric functions, $\ln(1 + x)$, $1/(ax+b)$ and $(1 + x)^n$.

Hyperbolic functions, higher order derivatives, Leibniz rule and its applications to problems of Type $e^{ax+b} \sin x$, $e^{ax+b} \cos x$, $(ax + b)^n \sin x$, $(ax + b)^n \cos x$, curvature, concavity and inflection points, asymptotes, Envelopes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital's rule, applications in business, economics and life sciences.

Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin^n x \, dx$, $\int \cos^n x \, dx$, $\int \tan^n x \, dx$, $\int \sec^n x \, dx$, $\int (\log \log x)^n \, dx$, $\int \sin^n x \, dx$, parametric equations, arc length, arc length of parametric curves, area and volume of surface of revolution.

Functions of several variables, limit and continuity of functions of two variables Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability. Chain rule for one and two independent parameters

6CC-MT-05: Differential Equation

Differential equations and mathematical models. General, particular, explicit, implicit and singular solutions of a differential equation. Clairaut's equations, Exact differential equations and integrating factors, separable equations and equations reducible to this form, linear equation and Bernoulli equations, special integrating factors and transformations

Lipschitz condition and Picard's Theorem (Statement only). General solution of homogeneous equation of second order, principle of super position for homogeneous equation, Wronskian: its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters

Systems of linear differential equations, types of linear systems, differential operators, an operator method for linear systems with constant coefficients,

Basic Theory of linear systems in normal form, homogeneous linear systems with constant coefficients: Two Equations in two unknown functions.

BVP & Eigen-value problems

Power series solution of a differential equation about an ordinary point, solution about a regular singular point.

6CC-MT-06: Riemann Integration and Series of Functions

Riemann integration; inequalities of upper and lower sums; Riemann conditions of integrability.

Riemann sum and definition of Riemann integral through Riemann sums; equivalence of two definitions; Riemann integrability of monotone and continuous functions, Properties of the Riemann integral; definition and integrability of piecewise continuous and monotone functions. Intermediate Value theorem for Integrals; Fundamental theorems of Integral Calculus.

Improper integrals; Convergence of Beta and Gamma functions. Differentiation under the sign of integration.

Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions. Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test.

Limit superior and Limit inferior. Cauchy Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass Approximation Theorem.

6CC-MT-07: Group Theory

Symmetries of a square, Dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups.

Subgroups and examples of subgroups, centralizer, normalizer, center of a group, product of two subgroups.

Properties of cyclic groups, classification of subgroups of cyclic groups. Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.

External direct product of a finite number of groups, normal subgroups, factor groups, Cauchy's theorem for finite abelian groups.

Group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms, First, Second and Third isomorphism theorems.

7CC-MT-08: Multivariate Calculus and PDE-I

Directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes, Extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, Definition of vector field, divergence and curl

Double integration over rectangular region, double integration over non-rectangular region, Double integrals in polar co-ordinates, Triple integrals, Triple integral over a parallelepiped and solid regions. Volume by triple integrals, cylindrical and spherical co-ordinates.

Change of variables in double integrals and triple integrals. Line integrals, Applications of line integrals: Mass and Work. Fundamental theorem for line integrals, conservative vector fields, independence of path.

Green's theorem, surface integrals, integrals over parametrically defined surfaces. Stoke's theorem, The Divergence theorem.

Partial differential equations of the first order, Lagrange's solution, nonlinear first order partial differential equations, Charpit's general method of solution, some special types of equations which can be solved easily by methods other than the general method.

Classification of second order linear equations as parabolic, hyperbolic or elliptic and their reduction to canonical forms.

Method of separation of variables.

7CC-MT-09: Ring Theory and Linear Algebra

Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring. Ideal, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals.

Ring homomorphisms, properties of ring homomorphisms, Isomorphism theorems I, II and III, field of quotients. Introduction to polynomial ring.

Vector spaces, subspaces, algebra of subspaces, dimension of sub-spaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces.

Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations. Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix.

Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators. Eigen spaces of a linear operator. Diagonalizability, invariant subspaces.

7CC-MT-10:Mechanics

Co-planar forces, Astatic equilibrium, friction, Equilibrium of a particle on a rough curve, virtual work, forces in three dimensions, general conditions of equilibrium, centre of gravity for different bodies, stable and unstable equilibrium.

Radial and cross-radial, Tangential and Normal components of acceleration, Equation of motion referred to a set of rotating axes.

Central forces, modeling of ballistics and planetary motion, Inverse Square Law, Kepler's laws on Planetary Motion.

Motion of a projectile in a resisting medium(including vertical direction, stability of nearly circular orbits, slightly disturbed orbits, varying mass. Motion of artificial satellites, constrained motion of a particle on smooth curve.

Degree of freedom, Moments and products of inertia. Momental ellipsoid, principal axes, D'Alembert's principle. Motion about a fixed axes. Compound pendulum. Motion of a rigid body in two dimensions under finite and impulsive forces, Conservation of momentum and energy.

7CC-MT-11: Probability and Statistics

Sample space, probability axioms, real random variables (discrete and continuous), cumulative distribution function, probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function, discrete distributions: uniform, binomial, Poisson, geometric, negative binomial, continuous distributions: uniform, normal, exponential.

Joint cumulative distribution function and its properties, joint probability density functions, marginal and conditional distributions, expectation of function of two random variables, conditional expectations, independent random variables, bivariate normal distribution, correlation coefficient, joint moment generating function (jmgf) and calculation of covariance (from jmgf), linear regression for two variables.

Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers, Central Limit theorem for independent and identically distributed random variables with finite variance, Markov Chains, Chapman-Kolmogorov equations, classification of states

Random samples, Sampling distributions, Estimation of parameters, Testing of hypothesis.

7CC-MT-12: Metric space & Complex analysis

Definition and examples of metric spaces. Open ball. Open set. Closed set as complement of open set. Interior point and interior of a set. Limit point and closure of a set. Boundary point and boundary of a set. Properties of interior, closure and boundary. Bounded set and diameter of a set. Distance between two sets. Subspace of a metric space

Convergent sequence. Cauchy sequence. Every convergent sequence is Cauchy and bounded, but the converse is not true. Completeness. Cantor's intersection theorem. \mathbb{R} is a complete metric space. \mathbb{Q} is not complete.

Continuous mappings, sequential criterion of continuity. Uniform continuity.

Extended complex plane, Riemann Sphere and Stereographic projection.

Functions of a complex variable, limit, continuity, differentiability, Cauchy-Riemann Equations, Sufficient condition for differentiability. Harmonic functions, Analytic functions, Bilinear transformation, Many-valued functions, Exponential functions, trigonometric functions, logarithm function.

Line integrals of a complex function, Cauchy's fundamental theorem and its consequences, Cauchy's integral formula. Maximum modulus theorem and its consequences, Morera's theorem, Liouville's theorem, Fundamental theorem of algebra.

Sequence and Series of Complex numbers and Complex functions, Uniform Convergence, Weierstrass' M-test, Weierstrass theorem on uniform convergence on Compact sets (statement only), term wise integration and differentiation. Power series, Cauchy-Hadamard theorem. Uniqueness theorem.

Taylor's theorem, zeros of an analytic function, form of an analytic function near a zero, zeros are isolated points.

7CC-MT-13:Real analysis-II

Outer measure on \mathbf{R} , Measurable sets, σ -algebra, Borel subset of \mathbf{R} , Lebesgue measure, Properties, Lebesgue measurable functions and its properties, Nonmeasurable sets, Cantor set, Pointwise & uniform convergence, Egoroff's theorem, Approximation by simple functions, Lusin's theorem.

Lebesgue Integration of nonnegative measurable functions, Fatou's lemma, Monotone convergence theorem, Integration of real valued measurable functions, Bounded convergence theorem, dominated convergence theorem, Comparison of Lebesgue and Riemann integral, Lebesgue criterion of Riemann integrability.

Vitali covering lemma, differentiation of monotone functions, differentiation of an integral, absolute continuity, fundamental theorem of integral calculus for Lebesgue integral.

Ring, σ -ring, algebra and σ -algebra of sets, Monotone class of sets, countably additive set function, measure and its basic properties, hereditary class, outer measure and measurability. Extension of measures. Complete measures and completion of a measure.

7CC-MT-14:Dynamical Systems

Dynamical System, Continuous dynamical System, Discrete dynamic system, Autonomous and non-autonomous dynamic system.

First order equations, existence, uniqueness theorem, Single species growth equation, logistic growth, Single species model with harvesting, Planar linear systems, equilibrium points, stability, Classification of equilibrium points, phase space, n-dimensional linear systems, stable, unstable and center subspaces.

Motion of pendulum, local and global stability, Liapunov method, periodic solution, Bendixson's criterion, Poincare Bendixson theorem, Gradient and Hamiltonian systems, limit cycle, attractors, index theory, Hyperbolic and non-hyperbolic equilibrium points, center manifolds.

Local Bifurcation of equilibrium points: Fixed points, saddle node, pitchfork, trans-critical bifurcation, Hopf bifurcation, co-dimension.

Logistic maps, equilibrium points and their local stability, cycles, period doubling, chaos, necessary conditions for chaos, Liapunov exponents, routes to chaos, tent map, Logistic map, horse shoe map. Deterministic chaos: Duffing's oscillator, Lorenz System.

7CC-MT-15:General Topology

Examples, Base for a Topology, Sub-base, Neighbourhood system of a point, Neighbourhood base, Limit point of a set, Closed sets, Closure of a set, Kuratowski closure operator, Interior and boundary of a set, Subspace Topology, First and Second Countable spaces.

Continuous functions over a Topological space, Homeomorphism, Nets, Filters, Their convergence, Product spaces, Projection function, Open and Closed functions, Quotient spaces, quotient maps, cone, cylinder, projective spaces.

T1,T2,T3,T4-Topological spaces, Completely regular spaces, Product of T2-spaces, Urysohn's Lemma in Normal spaces, Tietze extension Theorem, Embedding in cube. Embedding Lemma. Urysohn's metrization Lemma.

Open cover, Sub-cover, Compactness, Countable open cover, Lindeloff space, Compact sets, Finite Intersection property, Tychonoff Theorem on product of compact spaces, Continuous image of compact spaces, Locally compact spaces, One point compactification.

Connected sets, Separated sets, Disconnection of a space, Union of connected sets, Closure of a connected sets, Connected sets of reals, Continuous image of connected spaces, Topological product of connected spaces, Components, Totally disconnected spaces, Locally connected spaces.

8CC-MT-16:Algebra-II

Group actions, orbit of a point, stabilizer of a point, class equation and its applications, Cayley's theorem, properties of p-groups where p is prime, Cauchy's theorem, Sylow-p-subgroups and their properties, Sylow's theorem I, II & III, Simple groups up to order 60, Simplicity of A_n ($n \geq 5$). External and internal direct product of groups and their equivalencies, Fundamental theorem of finitely generated abelian groups.

Euclidean domain, PID, UFD, Polynomial ring over UFD, Polynomial ring over field, Irreducible polynomials and Eisenstein criterion, Finite fields, Finite field extensions using irreducible polynomials.

Inner product spaces, orthonormal basis, Quadratic forms, reduction and classification of quadratic forms, Equivalent Quadratic forms, Reduction of matrices to Diagonal or Normal Form, Jordan canonical form.

8CC-MT-17:PDE-II

Curves and Surface in three dimension, Orthogonal trajectories, Pfaffian Differential Equations and their solutions.

Classification, Existence of solutions, Integral curves, orthogonal surface, compatible system, Nonlinear PDE of 1st order, Characteristics method, Charpit's method, Jacobi method.

Linear PDE with constant coefficients, Cauchy-Euler equation, Classification, Riemann method of solution of linear hyperbolic equations.

Laplace's equation, Poisson equation, Boundary value problems, Laplace's equation in spherical polar and cylindrical co-ordinates, Heat equation, Harmonic function and Mean value theorem.

Diffusion equation. Boundary conditions, Solution by method of separation of variables, Maxima-Minima Principle, Uniqueness Theorem.

Wave equation and Helmholtz equation in spherical polar coordinates; Solution by the method of separation of variables: Solution by the method of Fourier series.

Green's function method for solution of Laplace's equation, Green's functions for solving Diffusion equation, Green's functions for Wave equation-Helmholtz theorem.

8CC-MT-18:Functional Analysis

Metric Topology, Complete metric spaces, examples. Separable metric spaces, Continuous functions; Homeomorphism, Isometry; Compact metric spaces, Sequential compactness, Heine-Borel theorem in \mathbb{R} . Finite intersection property, continuous functions on compact sets. Concept of connectedness and some examples of connected metric space, connected subsets of \mathbb{R} , \mathbb{C} , Banach Contraction Principle Theorem, Ascoli-Arzelà Theorem.

Banach space. $C[a, b]$ as a Banach space. Quotient space of a NLS, Algebra of convex sets. Bounded Linear operators, their continuity, Unboundable Linear operator, Norm $\|T\|$ of a bounded Linear operator T on NLS. Formulae for $\|T\|$. Equivalent norms, Riesz Lemma. Finite Dimensionality of NLS by compact unit ball, Boundedness of Linear operators over finite dimensional NLS, space $BdL(X, Y)$ of bounded Linear operators ; its completeness. Bounded Linear Functional, Hahn - Banach Theorem; its applications, conjugate spaces of NLS; Canonical mapping; Embedding of a NLS into its second conjugate spaces under a Linear Isometry; Reflexive Banach spaces; Open mapping Theorem: Closed Graph Theorem.

Cauchy - Schwarz inequality, I.P. spaces as NLS, Law of Parallelogram, orthogonal (orthonormal) system of vectors ; Hilbert spaces ; Projection Theorem in a Hilbert spaces H, Riesz representation for a bounded linear functional, Complete orthonormal system in H. Adjoint of bounded Linear operator in a Hilbert space H.

8CC-MT-19: Integral Equation and Integral Transform

Laplace Transform: Laplace of some standard functions, Existence conditions for the Laplace Transform, Shifting theorems, Laplace transform of derivatives and integrals, Inverse Laplace transform and their properties, Convolution theorem, Initial and final value theorem, Laplace transform of periodic functions, error functions, Heaviside unit step function and Dirac delta function, Applications of Laplace transform to solve ODEs and PDEs. Finite Laplace Transform: Definition and properties, Shifting and scaling theorem.

Fourier series: Trigonometric Fourier series and its convergence. Fourier series of even and odd functions, Gibbs phenomenon, Fourier half-range series, Parseval's identity, Complex form of Fourier series.

Fourier Transforms: Fourier integrals, Fourier sine and cosine integrals, Complex form of Fourier integral representation, Fourier transform, Fourier transform of derivatives and integrals, Fourier sine and cosine transforms and their properties, Convolution theorem, Application of Fourier transforms to Boundary Value Problems.

8CC-MT-20: Geometry of Manifolds

Manifolds: Topological and Smooth manifold, Chart, Atlas, Chart compatibility, Manifold with boundary

Smooth Maps on manifolds, Diffeomorphism, Examples, Partial derivatives, Inverse Function theorem and Implicit function theorem (statement only)

Tangent Space: Tangent space, The Differential of a map, Chain rule, Base for Tangent space, Computation in coordinates, Tangent bundle,

Curves in Manifolds: Definitions, Examples, Velocity vector of a curve, Definition of tangent space using curve, Computing the Differential Using Curves.

Submanifolds: Immersions and Submersions, Rank, and Critical and Regular Points, Definition of submanifold, Level Sets of a Function, The Regular Level Set Theorem, Examples of Regular Submanifolds.

Vector field: Definition, examples, Integral curve, Local flow, Lie bracket, Pushforward.

Lie groups: Definition, examples, Subgroup, homomorphism, Lie algebra.

Tensor: Multilinear Algebra, Symmetric and Alternating Tensors, Basic theory, Tensor field on manifold.

Differential forms: Differential of a function, Cotangent bundle, Pullback, Wedge product, Basic of Exterior derivative.

8CC-MT-21: Principles of Mechanics

Preliminaries: Concepts of Inertial frame, Newton's laws of motions, Conservative forces. Conservation laws. Equations of motion of a particle in different systems of co-ordinates. Motion of a particle on smooth and rough surfaces.

D'Alembert's principle: Constraints, Generalized coordinate, Energy Integral. Poisson Bracket and its properties. Poisson bracket relations concerning linear and angular momentum.

Action Principles: Hamilton's principle and the principle of least action, Verification of Hamilton's principle by D'Alembert's principle, Derivation of Lagrange's equations and Hamilton's equations from Hamilton's principle.

Symmetries and Constants of Motion: Noether's theorem, applications on important physical problems like Brachistochrone. Shortest distance, Laws of Reflection and Refraction.

Canonical transformation: Concept of Phase space, Different kinds of Canonical transformations. Configuration space. Point Transformation and equivalency of Lagrangian mechanics. Hamilton-Jacobi equation, application to action-angle variables.

8CC-MT-22: Graph Theory

Definitions, examples, walks, paths and cycles; length of a walk, closed walk, Circuits and cycles, Eulerian circuits, Hamiltonian cycles, Induced subgraphs, Degree Sequences, Havel-Hakimi Theorem (Statement only), Bipartite graphs and its basic properties, Isomorphism of graphs

Components of a graph; A simple Graph of n vertices and m components has at most $\frac{1}{2} (n-m)(n-m+1)$ edges ; Complete Graph, Complement of a graph; A Complete Graph of n vertices contains $\frac{1}{2} n(n-1)$ edges, Eulerian and Hamiltonian Graphs , Konigsberg Bridge problem, Separating set, (vertex) connectivity, connectivity of Hypercube graphs, disconnecting set, edge cut, edge connectivity, relation between vertex and edge connectivity, k - connected and k - edge connected graphs.

Definition of Tree, Important properties like A tree of n vertices has $(n-1)$ edges; A connected n - vertex graph with $(n-1)$ edges is a tree. Minimally connected tree; Spanning tree; Every connected graph has a spanning tree; Minimal spanning tree, Prim's and Kruskal's algorithms for finding a minimal spanning tree; Rooted tree, Binary tree.

Imbedding of a Graph on a surface, Faces of a Planar Graph; Euler's polyhedral equation, Subdivision of graph, Kuratowski's theorem (Statement only)

Adjacency matrix of a Graph, Incidence matrix of a Graph.

perfect matching, Hall's theorem, perfect matching in a k - regular bipartite graph, vertex cover.

Clique Number, Chromatic Number, upper bound using Greedy colouring and degree sequences, Map coloring, Fivecolor theorem.

8CC-MT-23:Advanced ODE & Special Functions

System of linear differential equations: System of linear differential equations in Normal form, Homogeneous linear system. Wronskian, Characteristic Equation and Characteristic Values, Stability of solution of ordinary differential equation.

Second order linear differential equations: Uniqueness Theorem, Characteristic Equation and Characteristic Values, Boundary Conditions. Sturm-Liouville Systems, Fourier's Convergence Theorem.

Green's function: Green's functions and its properties.

Special Functions: Equation of Fuchsian type, Series solution by Frobenius method, Bessel, Legendre, Hermite and Laguerre differential equations, Generating functions, Simple properties of solutions, Hypergeometric differential equations: Asymptotic Expansions, Solutions in terms of contour integration.

Numerical solutions of Differential Equations.

Minor Courses

NEC-MT-01: Basic Algebra

Polar representation of complex numbers, n^{th} roots of unity, De Moivre's theorem for rational indices and its applications. Exponential, logarithmic, trigonometric and hyperbolic functions of complex number.

General properties of equations, Descartes's rule of signs, relation between roots and coefficients, transformation of equations, cubic and biquadratic equations.

The inequality involving $AM > GM > HM$, Cauchy-Schwartz inequality.

Inverse of a matrix, Rank of a matrix,

Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation $Ax=b$, solution sets of linear systems, applications of linear systems, linear independence. characteristic equation of a matrix, Eigen value of a matrix, Eigen vectors, Cayley-Hamilton theorem.

Equivalence relations, Functions, Composition of functions, Invertible functions, One to one correspondence and cardinality of a set, Well-ordering property of positive integers, Division algorithm, Divisibility and Euclidean algorithm, Congruence relation between integers, Principles of Mathematical Induction, statement of Fundamental Theorem of Arithmetic.

NEC-MT-02: Fundamentals of Analytical Geometry

Techniques for sketching parabola, ellipse and hyperbola. Reflection properties of parabola, ellipse and hyperbola. rotation of axes, General Equation of second degree, classification of conics (including pair of straight lines) using the discriminant. Polar Equations of Conics, Tangent and Normal, Conjugate Diameters
Equation of planes, straight lines, Spheres, Cone, Cylinder, central conicoids, paraboloid, tangent and normals, planes section of conicoids, generating lines, classification of quadrics.

Introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions of single variable.

Vector Triple product, vector equation and application to geometry.

NEC-MT-03: Introductory Calculus

Limits of functions, Limit theorems, one sided limits. Infinite limits and limits at infinity. Continuous functions. Algebra of continuous functions. Continuous functions on an interval, intermediate value theorem,

Differentiability of a function at a point and in an interval, algebra of differentiable functions. Relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorem, intermediate value property of derivatives, Darboux's theorem. Applications of mean value theorem to inequalities and approximation of polynomials, Taylor's theorem to inequalities.

Cauchy's mean value theorem. Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder, application of Taylor's theorem to convex functions, relative extrema. Taylor's series and Maclaurin's series expansions of exponential and trigonometric functions, $\ln(1+x)$, $1/(ax+b)$ and $(1+x)^n$.

Hyperbolic functions, higher order derivatives, Leibniz rule and its applications to problems of Type $e^{ax+b}\sin x$, $e^{ax+b}\cos x$, $(ax+b)^n\sin x$, $(ax+b)^n\cos x$, curvature, concavity and inflection points, asymptotes, Envelopes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hospital's rule, applications in business, economics and life sciences.

Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin^n x \, dx$, $\int \cos^n x \, dx$, $\int \tan^n x \, dx$, $\int \sec^n x \, dx$, $\int (\log \log x)^n \, dx$, $\int \sin^n x x \, dx$, parametric equations, arc length, arc length of parametric curves, area and volume of surface of revolution.

Functions of several variables, limit and continuity of functions of two variables Partial differentiation, total differentiability and differentiability, sufficient condition for differentiability. Chain rule for one and two independent parameters

NEC-MT-04: Basic Linear Programming

Introduction to linear programming problem, graphical solution, convex sets, Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual.

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem, Travelling Salesman Problem.

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

Multidisciplinary Courses

NMD-MT-01- Statistical Techniques

Probability

Basic concepts, Classical definition of probability with its limitations, Axiomatic definition of probability, idea of random variables (with examples). Empirical and theoretical distribution with their properties, probability mass function and probability density function, mathematical expectation, conditional expectation. Variance and Co-variance.

Moments and moment generating function (mgf), properties of mgf, mgf of some distributions.

Markov chain, Chebyshev's inequality and its uses, Characteristic function and its properties.

Theoretical Distribution

Various discrete distributions e.g., Uniform distribution of the discrete type, Binomial, Negative Binomial, Poisson and Geometric distributions.

Various continuous distributions, e.g., Uniform distribution of the continuous type, Exponential, Erlangian, Gamma, Beta, Normal and Log-normal distributions.

Sampling distributions, e.g., Chi-square, t distribution, and F distribution and also their uses.

The idea of bivariate distribution, Bivariate normal distribution and its marginal and conditional distributions, Weak Law of Large Numbers (WLLN), Central Limit Theorem (CLT).

Survey Methodology

Sample Survey and Complete Enumeration, their advantages and disadvantages, sampling and non-sampling errors.

Different types of sampling, simple random sampling with replacement (SRSWR), simple random sampling without replacement (SRSWOR), idea of sampling errors in SRSWR and SRSWOR.

The method of drawing random samples, random numbers – their uses and properties, different tests for random numbers.

Random number generation using inverse transformation technique with reference to some standard distributions, e.g., Cauchy, exponential, gamma, etc.

Estimation Theory

Statistic and Parameter, properties of good estimator- unbiasedness, consistency, sufficiency and efficiency, with examples. The concept of completeness of a distribution. Basu's Theorem and its application.

Minimum variance unbiased estimator, Cramer-Rao Inequality and its uses.

The method of generating minimum variance unbiased estimator (MVUE), Rao-Blackwellisation. Examples with some standard distributions.

Method of Maximum Likelihood, Method of Moments.

Testing Statistical Hypothesis

Population and sample, Type 1 and Type 2 error, power of a test, level of significance of a test, uniformly most powerful (UMP) test.

Confidence co-efficient and confidence interval, point estimation and interval estimation, Confidence intervals for mean, variance and proportions.

Large sample theory of testing for mean, proportions. Chi-square test for goodness of fit.

Tests based on Chi-square, t and F – distributions.

Correlation and Regression

Association between two random variables, the idea of correlation co-efficient and its properties.

Mathematical relationship between random variables, regression equation, curve fitting by the method of least squares.

Regression equations considering the cases of two variables as well as three variables separately.

Partial and Multiple Correlation (for three variables only)

Skill-Enhancement Courses

NSE-MT-01: Analytical Geometry

Techniques for sketching parabola, ellipse and hyperbola. Reflection properties of parabola, ellipse and hyperbola. rotation of axes, General Equation of second degree, classification of conics (including pair of straight lines) using the discriminant. Polar Equations of Conics, Tangent and Normal, Conjugate Diameters

Equation of planes, straight lines, Spheres, Cone, Cylinder, central conicoids, paraboloid, tangent and normals, planes section of conicoids, generating lines, classification of quadrics.

Introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions of single variable.

Vector Triple product, vector equation and application to geometry.

NSE-MT-02: Linear Programming

Introduction to linear programming problem, graphical solution, convex sets, Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual.

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem, Travelling Salesman Problem.

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

NSE-MT-03: Computer Programming

Programming Language in C or any other language: Character set, Keywords, Basic data types, Numeric constants and variables operators, Expressions, Assignment statements, I/O – statements.

Control Statements: Decision making and Looping statements, break continue and goto statements, Example of simple programs. Subscripted variables: Concept of array variables in programming language, Rules for one dimensional subscripted variable, Simple programs.

List of Practicals (using any software)

- (i) Calculate the sum $1/1 + 1/2 + 1/3 + 1/4 + \dots + 1/N$.
- (ii) To find the absolute value of an integer.
- (iii) Enter 100 integers into an array and sort them in an ascending order.
- (iv) Bisection Method.
- (v) Newton Raphson Method.
- (vi) Secant Method.
- (vii) Method of False Position.
- (viii) LU decomposition Method.
- (ix) Gauss-Jacobi Method.
- (x) SOR Method or Gauss-Siedel Method.
- (xi) Lagrange Interpolation or Newton Interpolation.
- (xii) Simpson's rule, Weddle's rule(or Trapezoidal rule)

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